Main Research Interests

- Natural hazard statistics.
  - Landslides, wildfires, floods.
  - Implications for risk, erosion, ecology.
- Time series & spatial analyses.
  - Spectral analyses (e.g., wavelets), self-affine fractals (clustering, persistence, fractional noises).
- Models for examining above.
  - Cellular automata, self-organized behaviour, inverse-cascade models, etc.
- Confronting models with data.
- Heavy-metal contamination.
  - Wastewater and foodcrops in Zambia.

This Talk

- Aimed at non-experts.
- Introduce spatial & temporal scaling as related to nonlinear ‘geosciences’:
  - Basic concepts and some keywords.
  - Intuitive view of some techniques.
  - Examples.

Outline

1.0 Main Research Interests (1’)
2.0 Introduction (1’)
3.0 Scaling (Scale Invariance, Self-Similarity) (6’)
4.0 Scale Invariance Quantified (Power laws, Fractals, Ruler Technique, Box Counting) (10’)
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  5.3 Clustering in Time (10’)
6.0 Scaling: Issues to Consider (5’)

Nonlinear Sciences

- Application of nonlinear and/or stochastic differential equations to physical and social sciences.
Nonlinear Sciences

- Some key concepts/philosophies:
  - Fractals/multifractals – Chaos
  - Nonlinear waves – Solitons
  - Critical phenomena – Turbulence
  - Self-organizing systems – Pattern formation
  - Complex Systems – Cascades

- SCALING is an underlying attribute of many of the above.

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Scaling

- Can mean many different things.
- Comparison of ____ at small scales to large ones, or large to small.
- Downscaling, upscaling, multiscaling, …
- Fundamental difference between:
  – Characteristic scale vs.
  – Scale invariance

Characteristic Scale

Independent of scale. The shape does not define its size.
Scale Invariance

Rocks off Point Lobos, California

False-color topography of the Shanxi Province, China.

Self-similar:
Same pattern repeats itself at different scales.

Self-similar:
Construction of self-similar object (Sierpinski Gasket, Waclaw Sierpiński, 1915).

High voltage dielectric breakdown in plexiglass (Bert Hickman)
Self Similarity

Nature vs. Mathematical.

- Approximate vs. exact pattern.
- Range over which behaviour holds.

“For a stone, when it is examined, will be found a mountain in miniature. The fineness of Nature’s work is so great, that, into a single block, a foot or two in diameter, she can compress as many changes of form and structure, on a small scale, as she needs for her mountains on a large one.”

—John Ruskin (1819–1900)

Cast of Human Lung (Ewald R. Weibel)
Bronchial tubes branch off into smaller and smaller tubes, and are ‘self-similar’.
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Scale Invariance Quantified

• Benoît B Mandelbrot (b. 1924)
• Quantified scale invariance and self-similarity.
• Fractal: “A rough or fragmented geometric shape that can be subdivided in parts, each of which is (at least approximately) a reduced-size copy of the whole.”
• A fractal is, by definition, scale invariant, and exhibits power-law statistics.

Power-law statistics

• Underlying scale-invariance behavior is a power-law relationship.

\[ N_i \sim r_i^{-D} \]

- \( N_i \) = Number of objects
- \( r_i \) = Characteristic linear variable.
- \( D \) = Constant, the fractal dimension
**Power-law statistics**

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**Euclidean Dimension**

- A point has dimension \( 0 \)
- A line has dimension \( 1 \)
- A square has dimension \( 2 \)
- A cube has dimension \( 3 \)

**Fractal Dimension**


**Measuring D, Ruler Technique**

- **Measuring D, Box Counting**

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Bruce D. Malamud, Class Lecture UC Davis, 17 January 2008

\[ L(r) = 6150r^{-0.16} \]

- \( L(r) \): Length
- \( r \): Scale

\[ N(r) = 6150r^{1.16} \]

- \( N(r) \): Number of objects
Landslides Distribution
and Magnitude Scale
Scaling & Patterns in Nature
Box Counting Demonstration Program
http://www.fch.vutbr.cz/lectures/imagesci/harfa.htm

Number of boxes ($N$) as a function of Box Side Length ($r$)

$$\log(N) = -1.44 \log(r) + 4.10$$

$R^2 = 0.9998$

Measuring $D$, Box Counting

London’s urban boundary:
(a) 1888 (“A to Z” atlas)
(b) 2000 (Ordinance Survey map)
Small squares represent 1 km$^2$.

Coursework essay by J. Bridgen [AR/3031, Geography, KCL].
Measuring $D$, Box Counting

London’s urban boundary:
(a) 1888 (“A to Z” atlas)
(b) 2000 (Ordinance Survey map)

Coursework essay by J. Bridgen [AR/3031, Geography, KCL]. See also: work by Michael Batty [e.g. (2005) Cities and Complexity: Understanding cities with cellular automata, agent-based models and fractals].

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Time Series

- Set of numerical values of any variable that changes with time.
- [Can also take numerical values that changes in 1D (e.g., topographic profiles, well logs).]

Time Series

- Many attributes, including:
  - Frequency-size probability distribution of values. How many at a given size? [Ordering of values does not matter.]
  - ‘Pattern’ of the data series. Correlations (clustering) of values with each other. [Ordering matters.]

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Landslides Distribution

and Magnitude Scale

Scaling & Patterns in Nature
**Time Series**

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**Frequency-Size Statistics**

- **Earthquakes**
  - Small, medium & large earthquakes
  - Gutenberg-Richter scaling.
  - Power-law relationship between cumulative number of EQs ($N_{CE}$) and rupture area ($A_E$) the energy released.

  \[ N_{CE} \propto A_E^{0.89} \]

- **Landslides**
  - Medium and large landslides.
  - Frequency density as a function of area for 3 triggered event inventories.
  - Power-law (for medium and large landslides).

- **Wildfires**
  - Frequency-area distributions of medium and large wildfires.

  \[ f(A_r) \propto A_r^{\beta} \]

  \[ \beta = 1.31 - 1.49 \]

**Power-law statistics**

- Power-law graphs as straight line on log-log axes. Power-law exponent is slope of best-fit line, log-log coordinates.
Many natural hazards appear to satisfy heavy-tailed (e.g. power-law) frequency-size statistics.

- Earthquakes
- Wildfires (e.g., Malamud et al. [1998 Science; 2005 PNAS])
- Landslides and Rockfalls (e.g., Malamud et al. 2004, EPSL, ESPL)
- Volcanic Eruptions (e.g., Pyle 2000, Enc. Volcanoes)
- Asteroid Impacts (e.g., Chapman 2004, EPSL)
- Floods (e.g., Turcotte & Greene 1993 Stoch. Hydro. Hydraul.; Malamud and Turcotte J. Hydro. 2006)

Characterizing wildfire regimes in the USA
(with James Millington & George Perry, PNAS, 2005)

- USDA Forestry Service (USFS)
- 88,916 wildfires
- $A_F \geq 0.004 \text{ km}^2$
- 1970 – 2000
- Bailey ecoregion divisions: [common characteristics of soil, climate, vegetation]

In each ecoregion, what is frequency-area distribution of wildfires?

\[ \hat{f}(A_F) = \alpha A_F^{-\beta} \]
\[ \beta = 1.81 \]

\[ \hat{f}(A_F) = \alpha A_F^{-\beta} \]
\[ \beta = 1.30 \]
Characterizing wildfire regimes in the USA
(with James Millington & George Perry, PNAS, 2005)

Wildfire Statistics:
- # of big vs. small

Best-fit power-law exponents $\beta$ for all conterminous USA ecoregions.

Wildfire Probabilistic Hazard
- Wildfire recurrence interval $[T \geq A_F]$
  (Average time between events with burned area $A_F \geq 10 \text{ km}^2$, occurring in 1,000 km$^2$ region within ecoregion division; assumes ‘independence’ of events.)

Wildfire Frequency-Size Distributions: Some Questions
- If scaling, over how many orders?
- What is smallest scale useable for method?
- How do distributions vary due to human activity (e.g. population density)?
- How do distributions vary due to biophysical factors (e.g. biomass type, annual biomass produced)?
- What theoretical questions (e.g., ecological) are raised by heavy-tails?

Frequency-size distributions
- Many other examples with heavy-tailed (power-law) frequency-size distributions.
- Both social & physical sciences.
- Pareto Distributions (income, wealth), Zipf’s law (population, words), etc.

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Time Series
- Many attributes, including:
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Correlations (clustering) of values with each other. [Ordering matters.]
Pattern approximately repeats itself at different scales.

But we need to be careful, as x and y axes scale differently.

Pattern approximately repeats itself at different scales.

But we need to be careful, as x and y axes scale differently.

We call this a Self-Affine Fractal

Self-Affine Fractal

- Recall: $D =$ fractal dimension.
- ‘Smooth’ time series approach a straight line [$D \approx 1$]. [Low frequencies dominate over high ones.]
- Very ‘rough’ time series becomes area filling [$D \approx 2$]. [Large high-frequency component.]
- Time series are constrained: $1 \leq D \leq 2$. 
**Self-Affine Fractal**

**Formal definition**

\[ f(rx, r^{Ha}y) \]

is statistically similar to

\[ f(x, y) \]

\( Ha = \) Hausdorff exponent

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**Self-Affine Fractal**

- Many ways to measure the strength of self-affinity of a fractal (e.g. \( D \)), including:
  - Autocorrelation
  - Semivariogram Analysis
  - Hurst Rescaled-Range (R/S) Analysis
  - Fourier Power-Spectral Analysis
  - Wavelet Spectral Analysis
  - Detrended Fluctuation Analysis (DFA)
  - Inter-event occurrence times.
- Many others including lots of derivations of the above.

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**Fourier Power Spectral Analysis**

\[ S \sim f^{-\beta} \]

\( S = \) power spectral density

\( f = \) (Amplitude of Fourier coefficients)\(^2\)

\( \beta = \) frequency

\( \beta = \) constant

\( [\beta = 5 - 2D] \)

- \( \beta = 0 \) White noise. [Uncorrelated]
- \( \beta = 1 \) Pink (1/\( f \)) noise. [Medium strength long-range persistence.]
- \( \beta = 2 \) Brownian motion. [Strong long-range persistence, e.g. topography.]

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**Self-Affine Fractal**

- Most of these techniques examine the strength of the fluctuations of the data series as function of frequency (or wavelength).
- If the fluctuations are scale-invariant, it is a self-affine fractal.

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**Data Series**

- \( S \sim f^{0} \) (\( \beta = 0 \))

- \( S \sim f^{-2} \) (\( \beta = 2 \))
Power-spectral density of local atmospheric temperature from instrumental data and inferred from ice cores, time scales of 200 ky to 2 dy. [Pelletier and Turcotte, 1999]

Examples of ‘breaks’ in scaling behavior

Scaling

• Sometimes ‘breaks’ are more interesting than the same self-affine behaviour.
• Might indicate a change in underlying processes/physics.

Persistence

• Persistence: Correlations between adjacent values. Clustering of values (big follow big, small follow small).
• Short-range persistence: Values in the series affect only those close by.
• Techniques: AR [Autoregressive], MA [Moving Average], ARMA.

• Persistence: Correlations between adjacent values. Clustering of values (big follow big, small follow small).
• Long-range persistence: Values in the series affect those far away in time (space).
• Also called long memory, long-range correlations, etc.
• By definition, a self-affine fractal is one that exhibits long-range persistence.

Fractional Gaussian Noises created by Fourier Filtering

\[ S \sim f^{-\beta} \quad (S = \text{power spectral density}; f = \text{frequency}; \beta = \text{constant}) \]

\[ \beta = \{0, 1.25, 1.5, 2, 2.5\} \]

\[ \text{Hurst Rescaled-Range (R/S) Analysis} \]

\[ Q(t) = \text{Flow of water.} \]
\[ V(t) = \text{Volume of water in reservoir at a given time } t. \]
\[ V(T) = \text{Max and Min volume of water during a period } T. \]
Wavelets
Both spatial and frequency information.
Fourier Transform gives only frequency.
Wavelets: Especially suited for nonstationary signals.

Wavelet analysis of annual tree-ring widths for bristle cone pines (pinus longaeva) on Campito Mtn, California (ITRDB: USA; 37N,118W; Graybill & LaMarche; See also papers by Harlan; Ferguson.)

Wavelet variance analysis of annual tree-ring widths for bristle cone pines (pinus longaeva) on Campito Mtn, California (ITRDB: USA; 37N,118W; Graybill & LaMarche; See also papers by Harlan; Ferguson.)

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Wavelet variance analysis of annual tree-ring data for Campito Mtn, California (2111 BC to 1969 AD). Larger $\beta$ indicates stronger long-range persistence.

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**Scaling: Items to keep in mind (1 of 3)**

- What are lower/upper limits of scaling?
- What technique was used to arrive at the ‘measure’ of scaling?
- Some self-affine or long-range persistence techniques perform very poorly under the following conditions:
  - Very few data values.
  - ‘non-Gaussian’ probability distributions (particularly heavy-tailed).

**Scaling: Items to keep in mind (2 of 3)**

- Where is the natural sciences community headed with issues of scaling?
- If self-affine, does this indicate some sort of universality? [e.g. criticality, or self-organized criticality].
- Lots of disagreement.

**Scaling: Items to keep in mind (3 of 3)**

**Models vs. Data vs. Equations**

- Slider-Block Model
- Cluster Model
- Inverse-Cascade Model
- Forest-Fire Model
- Sand-Pile Model

**Questions? Comments?**

“Clouds are not spheres, mountains are not cones, coastlines are not circles, and bark is not smooth, nor does lightning travel in a straight line.”

—Benoit Mandelbrot (b. 1924)