Chaos and Fractals : Finding Hidden Order

A web page for Witt Sem 100 Fall 2005

"The scientist does not study nature because it is useful; he studies it because he delights in it and he delights in it because it is beautiful. If nature were not beautiful, it would not be worth knowing, and if nature were not worth knowing, life would not be worth living" - Henri Poincare (1854 - 1912)

"A mathematician who is not also something of a poet will never be a perfect mathematician" Karl Weierstrass (1815 - 1897)

"Now as Mandelbrot points out [...] .nature has played a joke on the mathematicians. The 19th century mathematicians may have been lacking in imagination, but nature was not. The same pathological structures that the mathematicians invented to break loose from 19th-century naturalism turn out to be inherent in familiar objects all round us in nature." -Freeman Dyson, Characterizing irregularity, Science 200(1978) 677-678.

"Clouds are not spheres, mountains are not cones, coastlines are not circles, and bark is not smooth, nor does lightning travel in a straight line ... Nature exhibits not simply a higher degree but an altogether different level of complexity". Benoit Mandelbrot

"The most exciting phrase to hear in science, the one that hearlds new discoveries is not 'Eureka' (I found it) but 'That's funny ..'" -Isaac Asimov

Course Description

"One of the great revolutions in science and mathematics in the last century was the realization that the universe is not always the predictable, clockwork place that it was thought to be. The new insights of fractal geometry and chaos theory that came out of this revolution have affected not only science and mathematics, but also fields as diverse as music, computer graphics, and economics. This course is an introduction to the ideas of fractals and chaos. Using these ideas, we will enter into a world in which simple rules give rise to almost unimaginable complexity and a fantastic and delicate beauty. Along the way we'll investigate questions such as "Can a butterfly flapping its wings in Brazil really cause a tornado in Ohio?" and explore the infinite levels of detail of the most famous fractal, the Mandelbrot set. As a final project, you'll be able to apply these concepts and techniques to an area of interest to you. You might choose to create a fractal musical composition, investigate why a perfectly regular heartbeat might not be as healthy as a somewhat irregular one, or study how chaos can be used in cryptography. This seminar meets the math reasoning requirement (M). "

Instructors

Professor Elizabeth George - Department of Physics - egeorge@wittenberg.edu Professor Brian Shelburne - Department of Mathematics and Computer Science - bshelburne@wittenberg.edu

Note : This site is *always* under-construction. As new and interesting sites are found over the summer, they will be added below

- Fractal Geometry website at Yale University sponsored by Michael Frame, Benoit Mandlebrot, and Nial Neger.
- <u>A Panorama of Fractals and Their Uses</u>: by Michael Frame and Benoit B. Mandelbrot
- Java Applets for Exploring Fractals: written by Ginger Booth and designed by Michael Frame.
- The Chaos Game: an introduction to the Chaos Game
- Dynamical Systems and Technology Project at Boston University: This project is a National Science Foundation sponsored project designed to help secondary school and college teachers of mathematics bring contemporary topics in mathematics (chaos, fractals, dynamics) into the classroom, and to show them how to use technology effectively in this process. It is directed by Robert L. Devaney of the Department of Mathematics at Boston University. At this point, there are a number of Java applets available at this site for use in teaching ideas

concerning chaos and fractals. There are also several interactive papers designed to help teachers and students understand the mathematics behind such topics as iterated function systems (the chaos game) and the Mandelbrot and Julia sets.

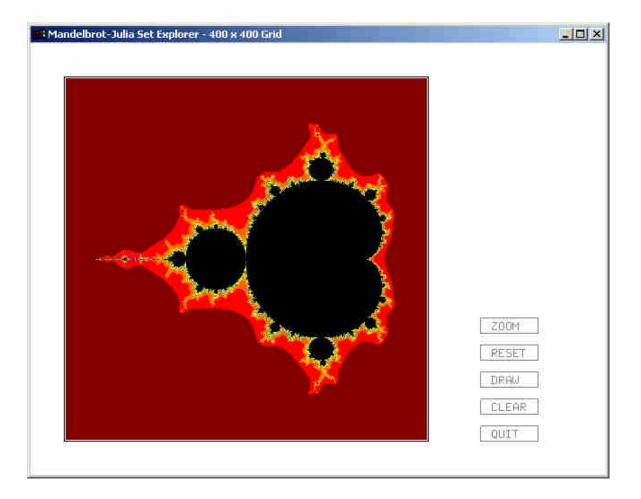
- 1. Chaos in the Classroom : A link to Devaney's web page on the Chaos Game
- 2. <u>A Chaos Game Applet</u>
 : Created by Johanna Voolich & Robert Devaney, this is a game where you try to place a point in the Sierpinski Triangle in a minimal number of moves. Be patient, it takes a little time to load!
- <u>Benoit Mandelbrot's personal web page</u>. Professor Mandelbrot is Sterling Professor of Mathematical Sciences at Yale University and IBM Fellow Emeritus, TJ Watson Research Center
- Arcadia:

Tom Stoppard's play weaves mathematics, fractals, and chaos together in a play that travels back and forth between the early 19th century and today. There are a number of good sites to check out. Try Skidmore College's web site on <u>Arcadia</u> or Devaney's web site <u>Chaos, Fractals and Tom Stoppard's Play *Arcadia*</u>. Another link to check out is a review from the November 1995 Notices of the AMS titled <u>Love and the Second Law of Thermodynamics: Tom Stoppard's *Arcadia*</u>.

- Chaos Under Control: The link to Peak & Frame's web site for their text Chaos Under Control.
- Fractal Coastlines A map that displays the computed fractal dimensions of the U.S coastline
- Verhust Population Dynamics! In the mid 19th century the Belgian mathematician <u>Pierre Verhulst</u> developed a population growth model (the <u>Verhust Equation</u>) which can be used to demonstrate chaotic behavior.
- There are lots of excellent web sites on fractals just go to <u>Google</u> and enter the word "fractal" for your search. <u>Sprott's Fractal Galley</u> is one site that immediately pops up.
- Fantastic Fractals Online contains downloadable soft ware.
 - 1. check out their stuff on Fractal Music
- <u>Metamedia's Fractals & Chaos Site</u>: this "new age" site has links to some good software
- The Game of Life: A Java Applet that plays Conway's Game of Life. This one is not to be missed!
- Java Fractals: This site contains a lot of Java applets for various fractals. Not too bad but wait for the applets to load
- Diffusion Limited Aggregation : Randomly moving particles colliding with other particles form aggregates with fractal-like structures
 - 1. DLA Applet #1
 - : A Java applet authored by Chi-Hang Lam at Hong Kong Polytechnic University demonstrates diffusion limited aggregation..
 - 2. The <u>Center of Polymer Studies at Boston University</u> has an excellent web page on Diffusion Limited Aggregation. In particular it has an <u>DLA Applet</u> which include the ability to change particle movement from random to straight.
 - 3. Another way to learn about Diffusion Limited Aggregation is to go to <u>Google</u> and search "Diffusion Limited Aggregation". See what pops up!
- Properties of Chaos

- a presentation demonstrating sensitivity to initial conditions, mixing, and density of periodic points in the Sawtooth and Tent functions, all of which are properties of chaos. Since this requires using binary representations of fractions, here is a <u>link</u> to a useful document

- Julia Jewels: An Exploration of Julia Sets by Michael McGoodwin - a thoroughly readble and enjoyable introduction to Julia Sets; a bit technical in some places but well worth the read; great links to other sites & software - in particular to the Chapter 2.2 of the Chaos HypertextBook by Glenn Elert (below).
- <u>The Chaos Hypertext Book</u> by Glenn Elert contains a number of highly readable and enjoyable articles on chaos. Check out <u>Chapter 2.2</u> on Julia Sets!
- <u>Fractals in Biology</u>: A University of Manitoba site which includes a long section on different methods of measuring fractal dimension.
- Construct Your Own Fourier Series: An Applet that lets you synthesize your own functions as a Fourier Series
- Sand Pile Applet : a demonstration of self-organized criticality (SOC) using a sand-pile model.
- Forest File Applet : another demonstration.of SOC.
- <u>MCell Home</u>: A site dedicated to Cellular Automata



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