Problem 4

Geology – Physics 30

1. Logistic Map

    a) Using the equation for the fixed point x* in terms of s, $x* = 1 - 1/s$, calculate the values for the fixed point x* for the following values of s:

    $$s = 1.0, 1.2, 1.5, 2.0, 2.3, 2.6, 3.0, 3.16$$

    b) Using the formula for absolute value of slope that we discussed in class, $slope(x) = s(1 - 2x)$, calculate the value for $|slope|$ at each of the fixed points found in a).

2. Thinking about the equation for the Second Return Map:

    $$x_{n+2} = F(x_n)$$

For the Logistic Map $x_{n+1} = s\, x_n\, (1 - x_n)$, do the following:

    a) The algebraic equation for the Second Return Map can be written as:

    $$x_{n+2} = s\, y_n\, (1 - y_n) \quad \text{where } y_n = s\, x_n\, (1 - x_n)$$

That is, first you calculate y_n from x_n , then from y_n you calculate x_{n+2} .

Use this equation to determine whether the 2 values $x_1 = .799456$, and $x_2 = .513044$ are fixed points for the Second Return Map for the value s = 3.20

    b) The slope of the second return map is given by the expression:

    $$slope = s^2 - 2\, s^2\, (s+1)\, x_n + 6\, s^3\, x_n^2 - 4\, s^3\, x_n^3$$

Calculate the absolute value of slope at the two points x1 and x2 in a). What do these values imply about the stability of these fixed points?