Statistical Distributions

Binomial

$$\binom{n}{x} p^{x} p^{n-x} = \frac{n!}{x!\,(n-x)!}\, p^{x} p^{n-x}$$

$\sigma^2 = np(1-p) \qquad \mu = np$ > Coin flips

Poisson

$$\frac{\mu^{x} e^{-\mu}}{x!}$$ > Counting

$\sigma^2 = \mu \qquad \mu = \mu$

Gaussian

$$\frac{1}{\sigma\sqrt{2\pi}}\, e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$ > IQ scores, Heights of people, Traffic accidents

μ, σ

Lorentz

$$\frac{\gamma}{\pi}\,\frac{1}{(x-\mu)^2+\gamma^2}$$

μ = mean
$\sigma \to \infty$ > stocks?

Log - Normal

$$\frac{1}{sx\sqrt{2\pi}}\, e^{-\frac{(\log x - M)^2}{2s^2}}$$

Daily Returns

$$\mu = e^{M + s^2/2}$$

$$\sigma^2 = e^{s^2 + 2M}\left(e^{s^2} - 1\right)$$

Inverse Gaussian or BPT

$$\sqrt{\frac{\lambda}{2\pi x^3}}\, e^{-\frac{\lambda (x-\mu)^2}{2x\mu^2}}$$

Kicked Oscillator

$$\mu$$

$$\sigma^2 = \frac{\mu^3}{\lambda}$$

Weibull

$$\left(\frac{\beta}{\tau}\right)\left(\frac{x}{\tau}\right)^{\beta-1} e^{-(x/\tau)^{\beta}}$$

Failures

$$\mathrm{CDF} = 1 - e^{-(x/\tau)^{\beta}}$$

$$\mu = \tau\, \Gamma(1 - 1/\tau)$$

$$\sigma^2 = \tau^2 \left[\Gamma(1 + 2/\tau) - \Gamma^2(1 + 1/\tau)\right]$$

$p(x) = \text{pdf}$

# of observations dN in $x, x+dx$:

$$dN = N\,p(x)\,dx$$

$$\mu = \langle x \rangle = \int x\,p(x)\,dx \quad \rangle \text{ continuous}$$

$$\mu = \langle x \rangle = \sum_{i=1}^{N} x_i \underbrace{\left(\frac{1}{N}\right)}_{P_i} \quad \rangle \text{ discrete}$$

$$\sigma^2 = \langle (x-\mu)^2 \rangle = \int (x-\mu)^2\,p(x)\,dx$$

$$\langle f(x) \rangle = \int f(x)\,p(x)\,dx$$

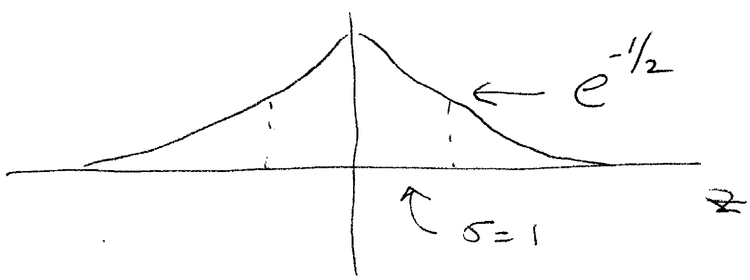
$$Z = \frac{1}{N} \sum_{i=1}^{N} x_i \qquad x_i \text{ random} \rightarrow \mu, \sigma$$

Z is a random deviate with mean μ, variance $\frac{1}{N}\sigma^2$

Gaussian Integrals

$$z = \frac{x-\mu}{\sigma}$$

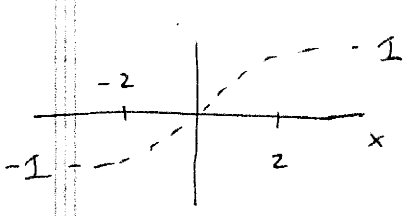
$$P_G(z) = \frac{1}{\sqrt{2\pi}}\, e^{-z^2/2}$$



Symmetric, skewness $= 0$

Error Function

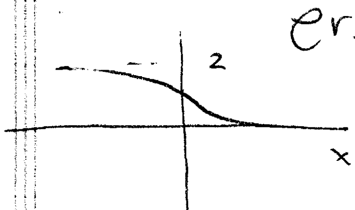
$$\operatorname{erf}(x) \equiv \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\, dt$$



$$= \frac{1}{\sqrt{\pi}} \int_{-x}^{x} e^{-t^2}\, dt$$

Complementary Scaled Error Fn:

$$\operatorname{erf}_c(x) = 1 - \operatorname{erf}(x)$$



Likelihood Function

$$L = \prod_{i=1}^{N} P_i \quad \rangle \text{ product of probabilities}$$

Log-likelihood:

$$\log L = \sum_{i=1}^{N} \log P_i$$

Gaussian Probability & Likelihood

If $P_i = p(x_i) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x_i-\mu}{\sigma}\right)^2}$

Then

$$L = \prod_{i=1}^{N} \left(\frac{1}{\sigma\sqrt{2\pi}}\right) e^{-\frac{1}{2}\left(\frac{x_i-\mu}{\sigma}\right)^2}$$

$$= \left(\frac{1}{\sigma\sqrt{2\pi}}\right)^N e^{-\frac{1}{2}\sum_{i=1}^{N}\left(\frac{x_i-\mu}{\sigma}\right)^2}$$

$$\log L = N \log\left(\frac{1}{\sigma\sqrt{2\pi}}\right) - \frac{1}{2}\sum_{i=1}^{N}\left(\frac{x_i-\mu}{\sigma}\right)^2$$

Useful in evaluating portfolios