Fourier Transforms

$$\varphi(q) = \int_{-\infty}^{\infty} p(x)\, e^{iqx}\, dx$$

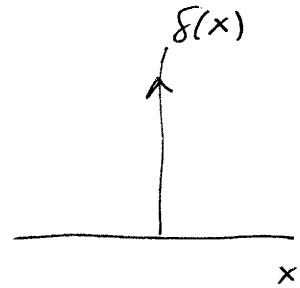
"Characteristic Function"

Also:

$$p(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \varphi(q)\, e^{-iqx}\, dq$$

Note:

$$\delta(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{iqx}\, dx$$



(Impulse fn.)

Properties:

$$\int dx'\, \delta(x-x')\, f(x') = f(x)$$

Application: all financial time series: price, returns, volatility

Then:

$$P(x) \stackrel{?}{=} \frac{1}{2\pi} \int_{-\infty}^{\infty} dq \int_{-\infty}^{\infty} dx' \, P(x') \, e^{iq(x-x')}$$

$$\stackrel{?}{=} \int_{-\infty}^{\infty} dx' \, P(x') \left[\frac{1}{2\pi} \int_{-\infty}^{\infty} dq \, e^{iq(x-x')} \right.$$

$$\stackrel{?}{=} \int_{-\infty}^{\infty} dx' \, P(x') \, \delta(x-x')$$

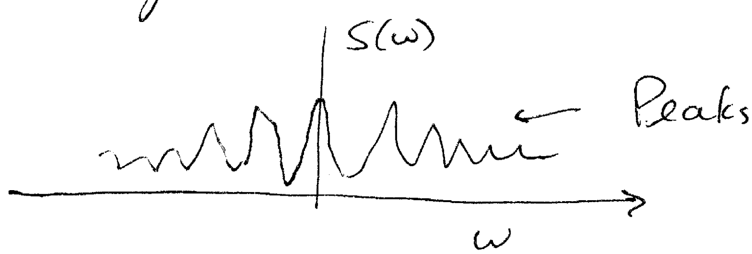
$$= P(x) \quad \checkmark$$

Note: If $q \rightarrow$ frequency ω

Then $S(\omega) = |\varphi(\omega)|^2 = \varphi(\omega) \cdot \varphi^*(\omega)$

is called the Power Spectral Density

It is symmetric around $\omega = 0$ (even fn)



It is easy to show that if $G(x)$ is a convolution of 2 functions $f_1(x)$ and $f_2(x)$:

$$G(x) = \int_{-\infty}^{\infty} f_1(x-y)\, f_2(y)\, dy$$

Then

$$\hat{G}(q) = \hat{f}_1(q) \cdot \hat{f}_2(q) \text{, where}$$

$\hat{G}(q)$ is the F.T. of G

$\hat{f}_1(q)$ is the F.T. of f_1

$\hat{f}_2(q)$ is the F.T. of f_2