BR 5-1

Monte Carlo Techniques

Idea is to determine the prob. distⁿ directly using the random number generator in a computer

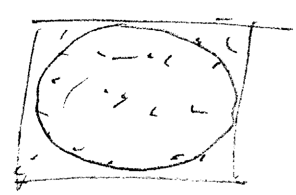
⇒ Hard to evaluate the convolution (chain) of probability densities

⇒ Evaluating a 1-d integral requires N function evaluations

$$2\text{-}d \iff N^2$$
$$3\text{-}d \iff N^3$$

Monte Carlo method provides means of simulating experiments

Example: Find area of circle of radius r_c. Construct a figure:



Throw grains of rice or particles. Count how many/fraction land in circle

N_c – land in circle
N_s – land in square = total

$$A_c = A_s \cdot \frac{N_c}{N_s}$$

This is the binomial distⁿ ⟷ "in" or "out" of circle. $p = \frac{A_c}{A_s}$ = prob of being "in" circle

Note – can't overpopulate square – "grains" of "rice" must be "dilute" & non-interacting (can't be bouncing off each other)

Uncertainty of area measurement
σ is determined from binomial distⁿ

$$\sigma = \sqrt{N_s p(1-p)} = \sqrt{N_c(1-p)}$$

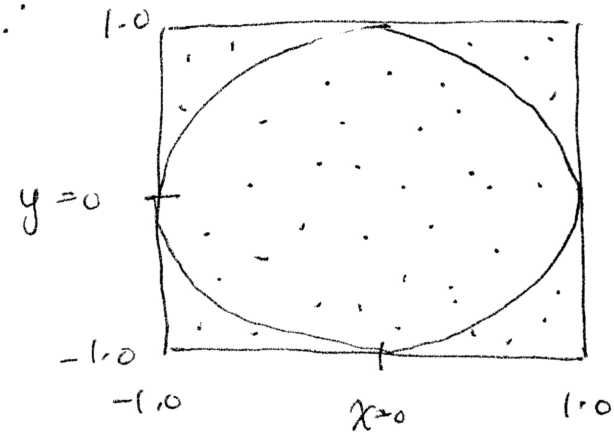
Relative error $\frac{\sigma}{N_c} = \frac{\sqrt{1-p}}{\sqrt{N_c}}$

So if $N_c \times 4$, $\frac{\sigma}{N_c} \downarrow \frac{1}{2}$

Replacing real rice grains by computer –

generated (pseudo) random numbers
is a big improvement

Experiment:



Compute 100 pairs of random #'s (x,y)
between -1 & 1. $N=100$

Prob of landing in circle is $p = \frac{\pi}{4} = \frac{\pi r^2}{4r^2}$

$= \frac{A_c}{A_s}$

In 100 tries $\mu = 100p = 78.5$

$$\sigma = \sqrt{Np(1-p)} = \sqrt{100(\pi/4)(1-\pi/4)} = 4.1$$

Thus we would expect to find

$$A_c = A_s \cdot \frac{N_c}{N_s} = \overbrace{(78.5 \pm 4.1)}^{N_c} \times \frac{\overbrace{2^2}^{A_s}}{\underbrace{100}_{N_c}}$$

tosses or grains out of 100 — tossed or grains out of 100 — # tosses or grains

We could re-write this as

$$A_C = \text{fraction} \times A_s$$

$$= \frac{(78.5 \pm 4.1)}{100} \times A_s = 2^2$$

$$= 3.14 \pm 0.16$$

Book - Fig 5.1, $A \simeq 2.92 \pm 0.18$

used experimental estimate of $p \sim \frac{73}{100}$ to estimate prob. & uncertainty

Book Fig 5.2 $\rightarrow$ 100 independent MC runs $\rightarrow$ $A = 3.127 \pm 0.156$
↳ of 100 tosses each

Random Numbers

We generate random #'s in the computer by means of a pseudo-random # generator. (not really random)

Any random # generator should satisfy the following criteria:

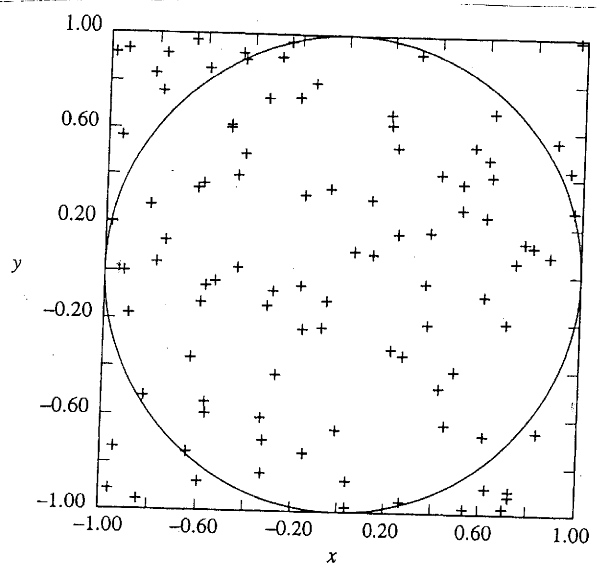


FIGURE 5.1
Estimation of the area of a circle by the Monte Carlo method. The plot illustrates a typical distribution of hits from one "toss" of 100 pairs of random numbers uniformly distributed between -1.00 and $+1.00$.

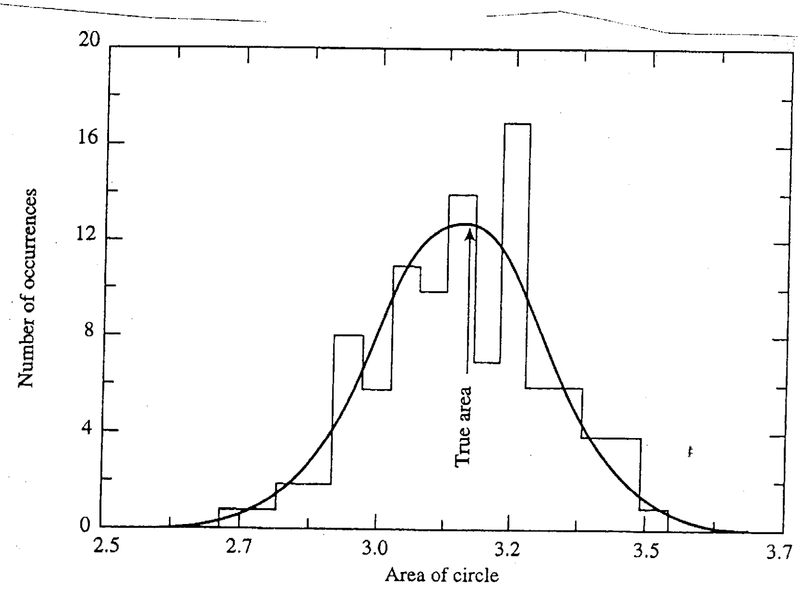


FIGURE 5.2
Histogram of the circle area estimates obtained in 100 independent Monte Carlo runs, each with 100 pairs of random numbers. The Gaussian curve was calculated from the mean $A = 3.127$ and standard deviation $\sigma = 0.156$ of the 100 estimated areas.