Scaling in Markets (Power laws)

No universally accepted model exists

Questions:

(a) Is 2^{nd} moment of price-change dist^n finite

(b) Is scaling observed?

(c) Over what time intervals (or other) is scaling seen?

Empirical Studies

M&S consider time evolution of S&P 500 over 6 yr period Jan 1984 – Dec 1989

Time series of index is $Y(t)$ – HF data

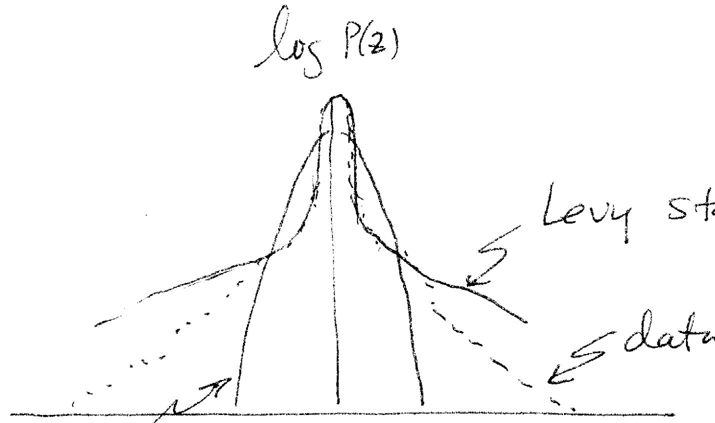
Compute pdf of

$$Z_{\Delta t}(t) = Y(t+\Delta t) - Y(t)$$

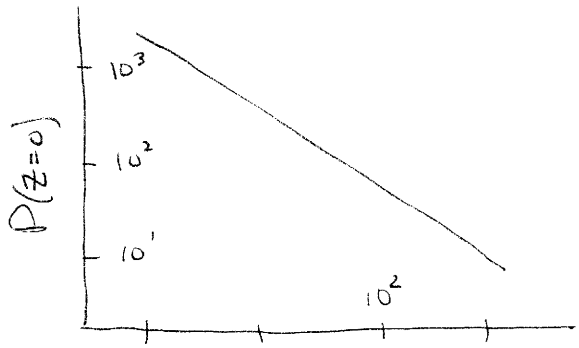
$\Delta t = 1\text{min}$ for the M&S data

pdf (see fig 9.1) is
- symmetric
- leptokurtotic
- non Gaussian for small changes

variance is finite (but can be large) since tails are not Levy!



Plotting prob of return $P_{\Delta t}(z=0)$ as a function of Δt we find



Scaling variables for Levy stable process

$$\tilde{z} \equiv \frac{z}{(\Delta t)^{1/\alpha}}$$

and

$$\tilde{P}(\tilde{z}) \equiv \frac{P(z)}{\Delta t^{-1/\alpha}}$$

A good value for $\alpha = 1.4$ for Levy

Properties :

a) A Lévy non-Gaussian scaling in central part of the dist^n

b) A Lévy non-Gaussian profile for $|z| \leq 6\sigma$

c) A finite variance (non-Levy)

Thus, scaling is observed that is not Gaussian, and not Levy.

Statistics of Rare Events

Cumulative prob of observing a change of size g or larger is

Fortune 1000: 1000 largest Companies 1994-1995

$$F(g) \sim g^{-\alpha} \qquad \alpha \approx 3$$

when $2 \leq g \leq 100$. Since $\alpha > 2$, 2nd moment of price changes is finite.

Correlation & Anti-Correlation

The basic idea here is to evaluate whether & by how much various financial time series are correlated. One goal of portfolio construction, for example, is to pick a set of uncorrelated investments.

Dynamics of pairs of stocks

For stock i, let $S_i = \ln Y_i(t) - \ln Y_i(t-1)$

so
$$\rho_{ij} = \frac{\langle S_i S_j \rangle - \langle S_i \rangle \langle S_j \rangle}{\sqrt{\langle S_i^2 - \langle S_i \rangle^2 \rangle \langle S_j^2 - \langle S_j \rangle^2 \rangle}}$$

correlation coefficient.

Here $Y_i(t)$ is the daily closing price of stock i at time t, and S_i is daily change of log of price of $Y_i(t)$.

$\langle \; \rangle$ = time average over all trading days during the period

Note that ρ_{ij} can range from -1 to 1, with the special meanings:

$$\rho_{ij} = \begin{cases} 1 & S_i \text{ completely correlated with } S_j \\ 0 & S_i \text{ uncorrelated with } S_j \\ -1 & S_i \text{ completely anti-corr. with } S_j \end{cases}$$

In M & S: Two examples – DOW 30 and S&P 500

1. For the DOW 30, there are $(30 \times 29)/2 = 435$ independent ρ_{ij}

Table 12.1

	Min	Max	
1990	0.02	0.73	among all 435 ρ_{ij} for each year
1991	−0.01	0.63	
1992	−0.10	0.63	
1993	−0.16	0.63	
1994	−0.06	0.51	

So stocks of the Dow tend to be correlated. (not really a surprise since those are biggest & most stable companies, and all are probably correlated with the economy at large)

We can also look at the time variation of the yearly correlations:

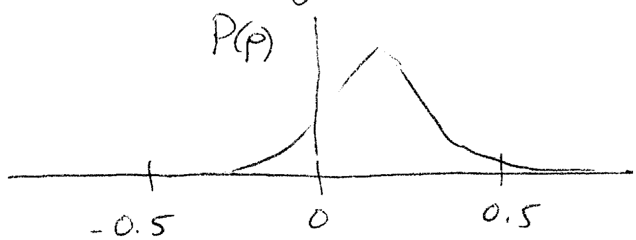
$$\delta_{ij} \equiv \frac{\rho_{ij} - \langle \rho_{ij} \rangle}{\sigma} \qquad (\text{M\&S } 12.4)$$

Deviation of ρ_{ij} from its average value using σ as the unit of measurement [presumably σ is the std. dev. of the ρ_{ij} values for the years]

S&P 500: 500 largest companies by market cap.

	Min ρ_{ij}	Max ρ_{ij}
1990	-0.30	0.81
1991	-0.29	0.74
1992	-0.25	0.73
1993	-0.27	0.81
1994	-0.25	0.82

General plot of $P(\rho)$ for SPX:



Note that a good (linear) way to analyze the ρ_{ij} is via principal component analysis (PCA).

Basically diagonalize the matrix ρ_{ij} to find the orthogonal (independent) constituents (eigenvectors)