Prediction for Earthquakes and Markets:
A Perspective from the Physics of Phase Transitions

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Abstract

Many complex systems in nature exhibit sudden changes in behavior or structure, which are direct consequences of the nonlinear dynamics of the system. These changes are often modeled as *first order phase transitions*, a phenomenon that was first investigated in the statistical physics community beginning with Landau and others over the last 100 years and more. The mathematical framework of this process is by now well established. In the recent past we have applied these ideas to earthquakes to gain a new understanding of the dynamics by which these damaging events occur. We now explore their applicability to the dynamics of financial markets, and show that a number of the commonly observed features of market behavior can be interpreted in terms of the dynamics of phase transitions. We also comment on the predictability of these changes.

Earthquakes and Markets

On July 31, 2010 the New York Times ran the story *A Richter Scale for the Markets* [Note 1] . This article was one of the first in the popular media to draw attention to recent research in the new discipline of *Econophysics* (Mantegna and Stanley, 2000), showing that earthquake dynamics may have important similarities to market fluctuations. But the basic idea is not new. Financial analysts have used the earthquake metaphor for markets frequently in the past.

Books such as Raghuram Rajan's *Fault Lines* (Rajan, 2010), Robert Reich's *Aftershock* (Reich, 2013), and Nouriel Roubini's *Crisis Economics* (Roubini, 2010) all illustrate the idea that economic and financial influences on markets are analogous to the forces driving tectonic plates, ultimately leading to earthquakes. Economic cycles that proceed from recession to expansion and back, and intermittent market corrections and crashes, are modeled by earthquake-like events. Market crashes are followed by a series of lesser corrections, which are seen as aftershock-like events.

A schematic illustration is shown in Figure 1a-c below. The state of the weak fault (dashed line) is shown in (a) at the start of the cycle, just after the last earthquake, with applied plate tectonic forces (arrows). In (b), increasing stresses are deforming the earth's crust and increasing the stress on the locked fault. In (c), the earthquake has just occurred, releasing the fault stress, and seismic waves are propagating outward from the fault.

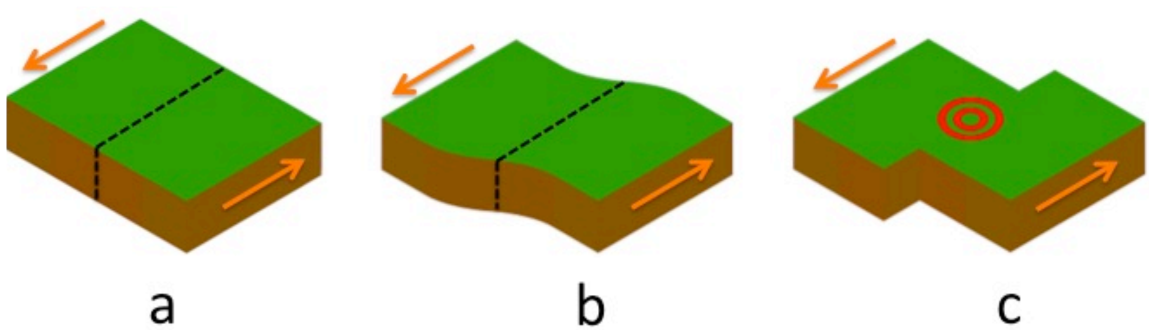


Figure 1. The earthquake cycle (Courtesy, Open Hazards Group)

In a more mathematical sense, the statistics of earthquakes and markets share striking similarities. For example, both exhibit the statistics of fat tails, as shown in Figure 2 below.

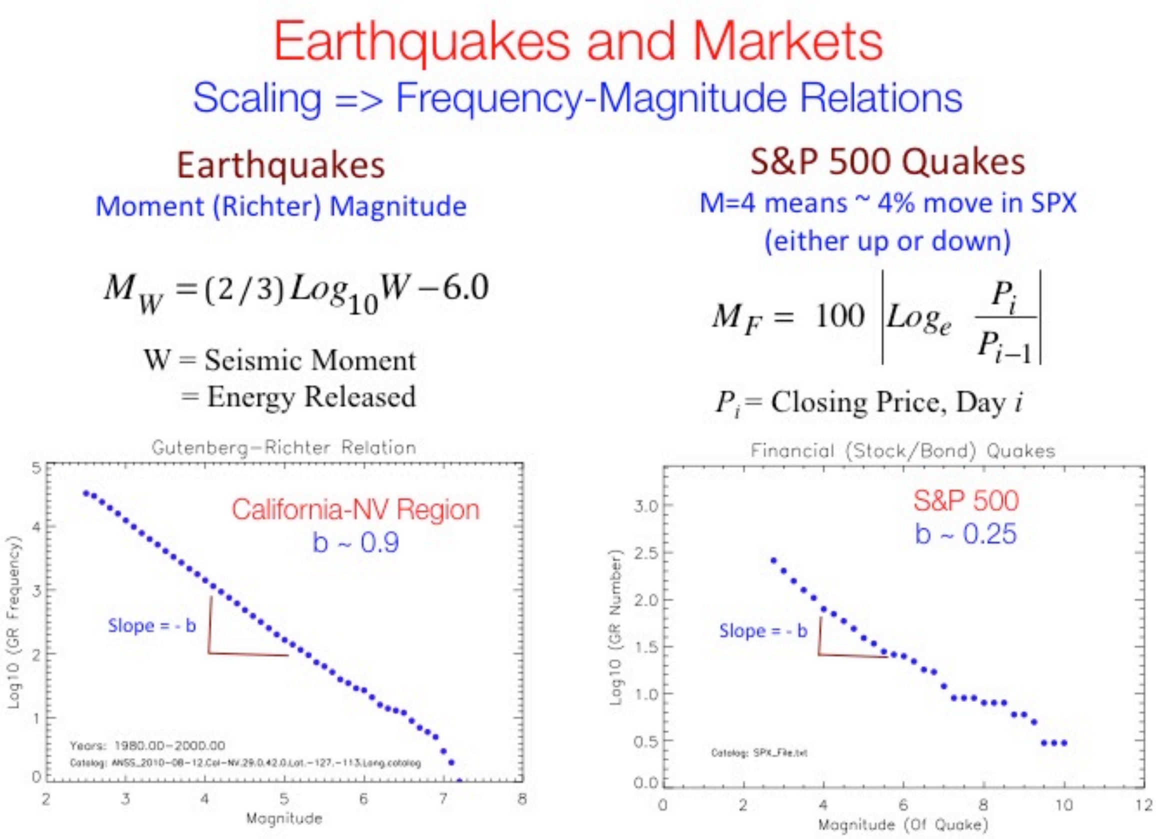


Figure 2: Fat tails in earthquakes and markets

Phase Transitions

How appropriate are these analogies and models? And more importantly, can we learn about markets by studying the dynamics of other systems that are subject to sudden large changes?

While the jury is still out on detailed answers to these questions, what we can say is that there are tantalizing similarities. In the type of physics studied here at the Santa Fe Institute in New Mexico, there is an important field of study focused on *phase transitions* [Domb, Gunton, Droz, Klein]. These are changes in the macroscopic state of a system as a result of external forces acting on the system.

We are familiar with one type of phase transition, usually called a *first order transition*. These occur, for example, when a bottle of liquid water is place in the freezer for several hours. The *liquid* phase transforms to the solid phase we call *ice*, in the process releasing a quantity of energy we call the *latent heat*. The water molecules still exist as individual particles, but instead of being in the macroscopic liquid state they have transformed as a group to the solid state.

In the refrigerator experiment, this phase transformation usually occurs gradually. But it is also possible to conduct the experiment in such a way that the transformation from water to ice occurs suddenly. It is now known that these same sudden transformations also characterize earthquake faults [Note 2; Rundle et al., 2003]. For this reason, earthquakes are now considered to be a type of first order phase transition of the earth's crust, from the intact state to the fully ruptured state.

This type of sudden change is much like a market crash [Note 3], which represents a sudden, macroscopic change in the character of the system. Phenomena such as hysteresis, persistence, correlations and states such as asset bubbles (or *meta-stable equilibrium*) are seen in both markets and earthquake systems as will be discussed below.

First order transitions are *non-equilibrium* transitions. They occur spontaneously and suddenly in response to external conditions, and once initiated, they cannot be controlled. They are said to be *irreversible* transitions because even if the external conditions are reverted to their original values, the system can remain for some time in the new phase.

Another type of phase transition is a *second order* transition. These are *equilibrium* transitions that occur gradually in response to slowly changing external conditions. They are reversible transitions. If the changes are subsequently reversed, the state of the system is reversed.

In second order transitions, and in some types of first order transitions, the changes in state are associated with the appearance of correlations in fluctuations of the system. For example, in second order transitions, there is a condition known as a *second order critical point* at which regions of all sizes are perfectly correlated. This has obvious analogies to markets, in which the growth and decay of correlations is associated with changing market conditions. A particular example in early 2016 [Notes 4,5] is the growth of significant correlation between the price of oil (WTI) and other commodities, FOREX levels, and the Standard and Poors 500 index. An interesting question is whether these correlations may explain the period of high market volatility seen in early 2016.

In the case of earthquakes, the events and their correlations are driven by plate tectonic forces. In the case of markets, the events and their correlations are driven by liquidity or money supply through the FOMC rate-setting policies, leverage, and general economic conditions [Peterson et al., 2010].

Stability

Phase transitions occur in a wide variety of physical, social, and engineered systems when nonlinear processes are present. In those contexts, they are also called *regime changes* or *tipping points*. This post will discuss some of the technical details of the theory of phase transitions and how these relate to the stability of earthquake fault systems and markets.

For purposes of illustration, let us assume that there exists a function $U[\phi]$ that describes the "cost" associated with creating a given state ϕ of a system. In physics, such a function is called a *free energy* function. In biology, the function $-U[\phi]$ is usually termed a *fitness function*. For markets, $-U[\phi]$ can be considered to be a utility function in the sense of Bernoulli [Note 6].

In physics, the variable ϕ is called an *order parameter* because it is associated with the evolution of the system from one state of order to another. The evolution may proceed suddenly, or perhaps via a series of random, chaotic, or intermediate states.

In physics, systems evolve to minimize the free energy or the cost. In biology, systems evolve towards maximum fitness. Likewise, we can consider that markets evolve to maximize utility. These considerations provide a context for the dynamics of these systems, and a rationale for their evolution in time.

To be more specific, we look at an example. We note in passing that the model we discuss here assumes that the system of interest is a *near mean field system*, which in a practical sense means that the *range of interaction* between the components of the system are *long range*. Specifically, each component or agent in the system interacts with many other agents simultaneously. In a practical sense, it can be argued that both earthquakes and markets fulfill this condition (Rundle et al., 2003; Klein et al., 2007; Mantegna and Stanley, 2000)

We define a cost function:

$$U[\phi] = -f\phi + \epsilon\phi^2 + \alpha\phi^4$$

Here, the function $U[\phi]$ describes the free energy or cost required to create the system state ϕ. In physics, this function is identified with the *Landau Theory* of phase transitions.

We further assume that α is a positive number, while f and ϵ can be either positive or negative. A plot of this function in four specific cases can be seen below.

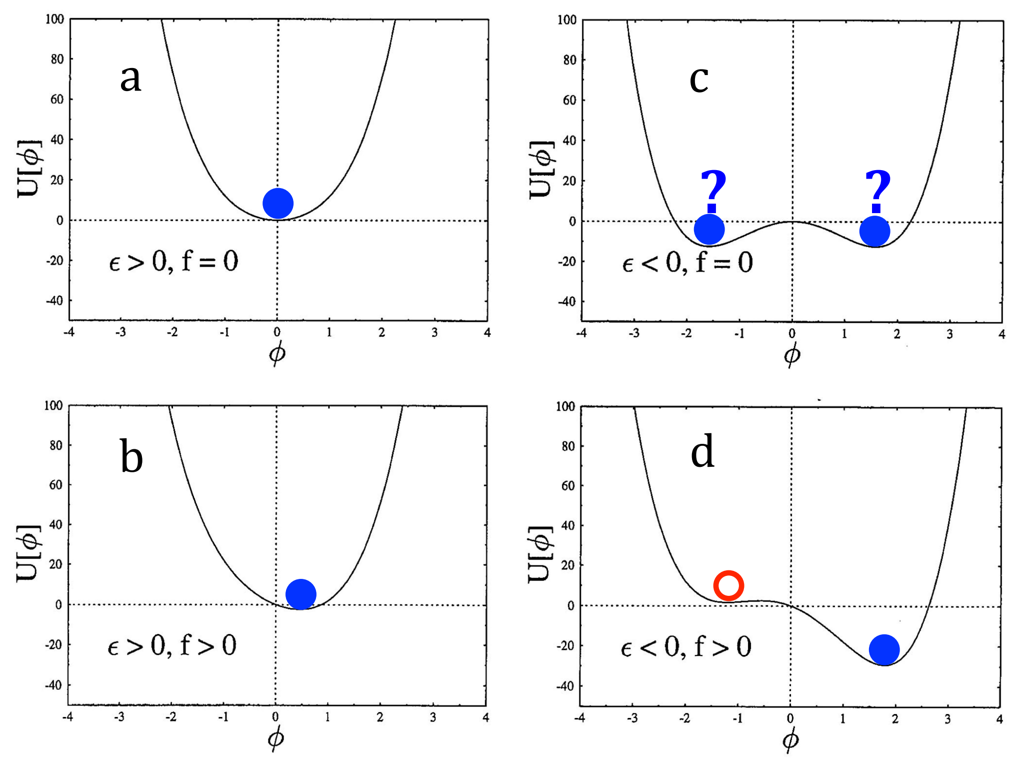


Figure 3. Free energy cost as a function of the order parameter ϕ, for different values of ϵ, f.

In figure *a*, there is only one minimum in the center at $\phi = 0$. The principle of minimum cost (minimum free energy) means that the system remains at the stable state $\phi = 0$ indicated by the blue dot. This is the only *equilibrium* state.

In figure *b*, the minimum state has been shifted to the right as f is increased, so the equilibrium state ϕ has a positive nonzero value. This is an example of an equilibrium transition, since in going from *a* to *b*, a continuous change in variable f is associated with a stable, reversible, continuous change in state ϕ. There is still just one equilibrium state.

In figure *c*, there are now two minima (two stable states) and the system could be in either of them. There are two possible equilibrium states, and the choice of which to occupy is determined by the details of evolution of the system. A *selection rule* of some sort would be needed to determine which state the system occupies.

Figure d shows the most interesting example. Now there are two minima, one of which is a *local* minimum (left hand state, red circle), and the other of which is the *global* minimum (right hand state, blue dot).

In accord with the principle of least cost or minimum free energy, the system state would normally correspond to the right hand state (blue dot). However, it is possible to imagine dynamical processes that lead to the system residing in the left hand, local minimum state (red circle). An example of such a process for a driven earthquake model has been described in Rundle et al. (1997).

States of *metastable* equilibrium can often be long-lived, a condition that John Maynard Keynes essentially described by the pithy statement that "the market can stay irrational longer than you can stay solvent". Phenomena associated with this state of *metastability* also include *hysteresis*, which we discuss below.

After some average period of time termed the *metastable lifetime*, the state of the system can transition suddenly and spontaneously from the left hand, higher energy state, to the right hand, lower energy state. This spontaneous transition in state is the model for an earthquake or market crash.

Conditions leading to the decay from metastablity are associated with tipping points. Here it is said that the system has *nucleated* to the state of lower energy. These nucleation events are the first order phase transitions described above.

Hysteresis

Hysteresis can be defined as the condition of a complex system in which the current state of the system tends to persist for a period after it has ceased to be the preferred, long-term stable state of the system. Peter Bernstein (2008) and Jared Bernstein (2016) have described the application of the idea of hysteresis to the financial markets and to more general economic systems.

Recall that metastability is the temporary state of a system that is different from the long-term, preferred state of the system. The idea of hysteresis is associated with metastability, but an important question relates to the process by which the system evolves into a metastable state. To answer this question, we return to the *Landau theory* of phase transitions described previously.

We defined the *order parameter* of the system, together with a cost or fitness function. The order parameter describes the current state of the system, while the cost function defines the dynamics by which the system evolves. The system is driven by a persistent forcing f. For an earthquake fault, f is the plate tectonic forces.

For markets, f is the liquidity or money supply. For earthquakes, convective circulations deep within the earth's mantle produce the tectonic forces that drive the plates. For markets, the federal reserve and other central banks supply the money necessary to grow the economy.

The Landau theory postulates that the system evolves towards a state of minimum free energy. This state can either be a local minimum, or a global minimum. An example for an earthquake fault is shown in the figure below , where there are two local minima. In this case, the order parameter is the deficit in slip on the earthquake fault.

Because the earthquake fault is locked between earthquakes, the slip state on the fault lags behind the far-field displacement of the plates as the tectonic forces increase. The slip deficit is defined as the local current slip on the fault minus the far-field plate tectonic displacement. Thus as the plates move between earthquakes, the slip deficit becomes increasingly negative. The elastic potential energy (free energy) stored by the fault also increases, much as the energy of a spring increases as it is stretched.

When the earthquake occurs, the potential energy is released, and the slip increases suddenly, meaning that the magnitude of the slip deficit decreases to zero. This process is illustrated in figures 4a and 4b, where figure 4a corresponds to the globally stable state, and figure 4b corresponds to the local metastable state. Hysteresis refers to the idea that the locked state of the fault tends to persist even after it is no longer the globally stable state, due to the potential energy barrier ("hill") separating the metastable minimum ("higher valley") from the lower, global minimum ("lower valley").

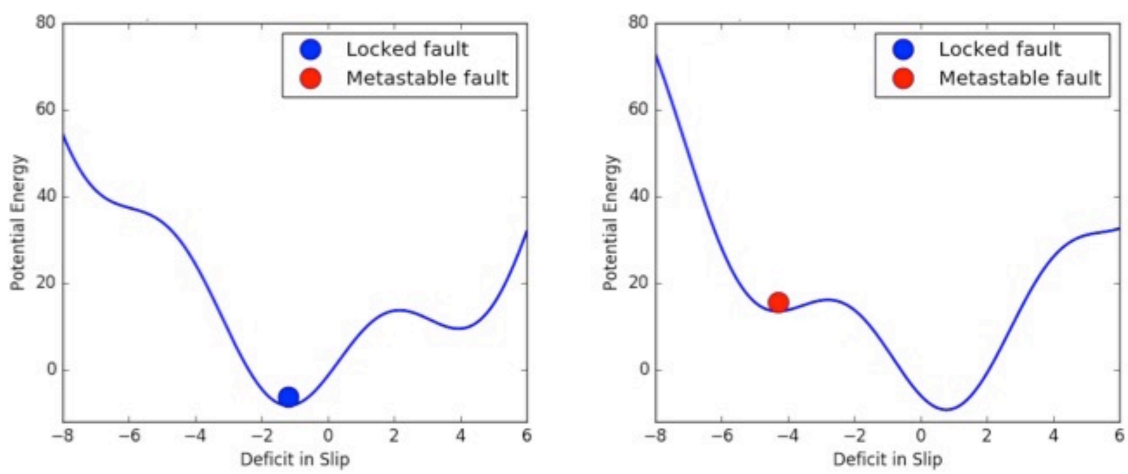
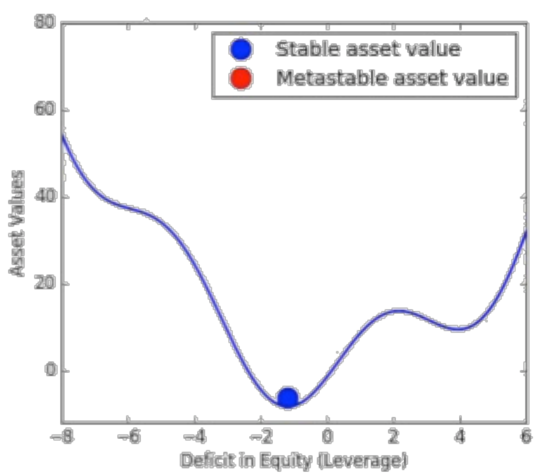


Figure 4a,b. Landau theory for potential energy of locked fault in the (a) stable configuration, and (b) in the metastable configuration.

The decay itself is ultimately caused either by random fluctuations ("noise") inherent in the system, or by a lowering of the free energy barrier by an external perturbation, which may happen as the dynamical state evolves in time. As the free energy barrier becomes smaller, the fluctuations have a greater chance of nucleating the transition. An important question relates to how the system evolves into the metastable state in the first place, a point that we discuss below. All these processes have been described mathematically in the statistical physics literature in great detail.

The same type of model can be applied to the markets, as shown in figures 5a and 5b. In this application, the earthquake cycle of "earthquake > locked fault > earthquake" is replaced by the business cycle of "recession > economic boom > recession", or the leverage cycle of "market crash > inflated assets > market crash". Here these economic systems would be driven by the increase in available liquidity arising from the increase in the money supply from the Federal Reserve, banks, the growth of GDP, and the velocity of money, among other factors.



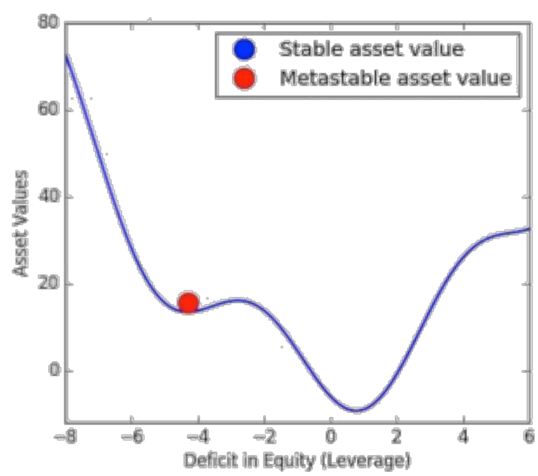


Figure 5a, b. Laudau theory for asset values as a function of leverage (deficit in equity) for the (a) equilibrium and (b) metastable (asset bubble) configuration.

In the case of the markets, growth in the money supply drives the market cycle because liquidity increases most rapidly after the crash has occurred, and tends to persist too long when it is no longer needed. Late in the cycle, as the Federal Reserve removes liquidity, asset values can no longer be supported at their then-present levels. As the financial system moves deeper into the metastable state, asset values rise beyond what they would otherwise be in the lowest value, equilibrium state. These inflated asset values correspond to the *asset bubbles* that are created when there is an excess of liquidity in the financial system.

In the view of economists such as John Geanokoplos (2010), moral hazard and other forms of risky behavior tend to grow throughout the market cycle, and in turn lead to an increase in leverage. Leverage is nothing more than an increase in the deficit in equity used to purchase an asset. Thus the increasing deficit in equity for markets corresponds to the increasing deficit in slip on earthquake faults leading up to the nucleation event (earthquake or market crash).

In the movies shown in [Note 7], we show examples of the evolution of the earthquake and market cycles. At the beginning, the system occupies the global stable minimum, indicated by the blue color of the circle. Under the dynamics, the system evolves to produce a higher, metastable minimum as well. The red circle indicates that the state is metastable. The metastable state eventually decays and the system transitions to the stable state once again. The cycle continues as the a new metastable state is formed, and the system is lifted to higher energies and higher asset values. Hysteresis is associated with the fact that the system remains in an elevated potential energy or asset-valued state long after it should have transitioned. Mathematical details can be found in Rundle et al. (1997).

Prediction

Given a seismically active local region, a problem of interest is to determine how much stress and strain has accumulated since the last major earthquake. In other words, we would like to determine the level of progress of the region through the "earthquake cycle". This earthquake cycle problem is complicated because the absolute stress and strain since the last major earthquake cannot be determined from direct observations at all locations of interest. However, a new method, "nowcasting", is available that may provide an answer to this problem.

Nowcasting is a term that arises in economics and finance in reference to the business cycle. It refers to the determination of the current uncertain state of the economy or the markets (Giannone, 2008; Banbura et al., 2013) as the business cycle progresses from recession to boom and back. Nowcasting is used to routinely monitor the state of the business cycle in real time. In economics, for example, it is critical for the Federal Reserve Board to know the gross domestic product (GDP) and how it is changing with time, so that an appropriate Fed funds rate can be chosen. The GDP is the monetary value of all the goods and services produced within a country during a specified period, and is a broad measure of overall economic activity. Determining the GDP is extremely difficult. In the US, the Bureau of Economic Analysis estimates the value of GDP from total consumption, expenditures, production, and trade balances among other variables (Banbura et al., 2013). In subsequent quarters, as more data becomes available, GDP is often revised and corrected for seasonal factors.

As a result of this uncertainty, the current value of GDP must be regarded as a random variable. Many recent papers have proposed that the GDP could be determined by indirect variables obtained from statistical analyses of economic time series data, and even social media data (Giannone et al., 2008). This is the nowcasting problem, the determination of an economic variable by indirect means.

Another example of the use of nowcasting is estimating the present amount of leverage, or debt in the financial markets. For this, one must account for the value of all monetary aggregates, as well as all contracts that involve financial agreements and obligations, such as reverse repurchase agreements, mortgages, brokerage margin accounts, credit default swaps (CDS), collateralized debt obligations (CDO), and so forth. Many of these financial instruments suffer from a lack of transparency. It has been estimated that the value of all financial derivatives, for example, is of the order of hundreds of trillions of US dollars (National Commission, 2011), but the exact value is uncertain.

In earthquake fault systems, earthquakes are observed to repeat within a seismically active region (Scholz, 1990). Determining the progression of the region through its earthquake cycle is important for estimating the level of seismic hazard. In the past, this determination of the state of a regional fault system has focused on trying to estimate the state of stress in the earth, its relation to the failure strength of the active faults in a region, and the rate of accumulation of tectonic stress (Scholz, 1990). Determining the values of these parameters would allow researchers to estimate the proximity to failure of the faults in the region. This would be an answer to the question of "how far along is the region in the earthquake cycle?". However, such a program of reseasrch is very difficult, because it is not possible to determine the state of stress and strain in general. As a result, the current state of the fault system remains uncertain and elusive.

Here we use the idea of nowcasting to answer this question. More specifically, we use the accumulation of small earthquakes since the last large earthquake in a defined region to estimate the current hazard level in the region. Event counts as a measure of "time", rather than the clock time, is known as "natural" time (Varotsos et al., 2011, Holliday et. al., 2006). We will show that the use of natural time has at least two advantages when applied to seismicity:

- It is not necessary to decluster the aftershocks. The natural time count is uniformly valid when aftershocks dominate, when background seismicity dominates, and when both contribute.
- Natural time statistics are independent of the level of the seismicity as long as the slope of the magnitude-log(frequency) distribution is approximately constant. In computing nowcasts, the concept of natural time, counts of small earthquakes, is used as a measure of the accumulation of stress and strain between large earthquakes in a defined geographic region.

Note that nowcasting, which describes the present state of a system, is distinct from the idea of forecasting, which looks forward in time (Holliday et al.,2016, Rundle et al., 2012). Nowcasting is the calculation of the current state of the system. Nowcasting can be used as a basis for forecasting if a method is used to project the current state into future states. In fact, nowcasting should be a prerequisite to forecasting, the estimation of the future state of the system. The current state must be known, at least approximately, before the future state can be accurately estimated.

In other publications, we have shown that these natural time methods, which rely on counting small events, can be used as a basis for forecasting in the Natural Time Weibull method (Rundle et al., 2012; Holliday et al., 2016). Here we use the count of small earthquakes to establish progress through the earthquake cycle in a region, then we project forward in time by assuming that the rate of small earthquake occurrence is defined by a Poisson rate. We then convert this future estimated rate of occurrence to a probability using a Weibull distribution. Finally, we test the forecast accuracy and reliability using standard testing procedures to optimize the parameters in the model (Rundle et al., 2012). Global earthquake forecasts using this method can be obtained from the Open Hazards web site [Note 8]

An example of several nowcasts are shown in the table below for several global megacities as of April, 2016. Data we used correspond to cycles of earthquakes having magnitude larger than 6.0, within 100 km of the designated city. The term "EPS" corresponds to the "Earthquake Potential Score", on a scale of 0% to 100%. The table shows that Santiago, Chile was at that time the city that had progressed the most through the earthquake cycle, being 86.3% towards the occurrence of the next M6 event.

City	**Region**	**Population (Millions)**	**Number of Regional Earthquake Cycles**	**EPS (Highest=100)**
Santiago, Chile 33.45^{o}S, 70.66^{o}E	57^{o}S to 18^{o}S 83^{o}W to 56^{o}W	5	313	86.3
Manila, Philippines 14.60^{o}N,120.98^{o}E	5^{o}N to 20^{o}N 116^{o}E to 129^{o}E	1.6	196	74.0
Tokyo, Japan 35.69^{o}N,139.68^{o}E	25^{o}N to 50^{o}N 125^{o}E to 155^{o}E	39	659	68.7
Taipei, Taiwan 25.o3^{o}N,121.63^{o}E	19^{o}N to 27^{o}N 117^{o}E to 124^{o}E	2.6	95	47.4
Jakarta, Indonesia 6.17^{o}S,106.82^{o}E	12.25^{o}S to 6.25^{o}N 90^{o}E to 115^{o}E	32	235	38.3

Table 1: EPS values for 5 global megacities

With respect to markets, it is not as yet clear whether or how these ideas on nowcasting and forecasting using small events could be applied to the financial markets. Yet it has been noted in the econophysics literature that volatility in prices reacts in much the same manner as earthquake aftershock seismicity to financial "shocks", such as sudden announced changes in the positions of the Federal Open Market Committee [Peterson et al., 2010]. There is thus reason for believing that many of the same types of prediction techniques may carry over directly.

Limits to Predictability

We can use the analogy to phase transitions, and the Landau theory discussed above, to understand the limits to predictability of both earthquakes and markets. The key is to understand the relevance of figures 3c and d. As the driving force, tectonic plate motion or increase in available liquidity, increases the system goes deeper into the metastable state. The deeper the system goes into the metastable state the more likely the first order phase transition will occur via the process of nucleation.

Another way to look at this idea is that the nucleation event has to overcome a barrier. When the two free energy minima are at the same depth, as in figure 3c, the barrier is infinite and either phase will last an infinite time. That is, they are both stable as we discussed above.(Gunton and Droz, 1983, Gunton et al., 1973)

As the driving force increases the barrier decreases and the time the system spends in the metastable state decreases. In very long range systems such as those that include earthquake faults and markets the system will reach a state in which the metastable well is shallow and almost disappears. We refer to the point at which the well disappears as the spinodal. The spinodal is a critical point and has the characteristics we described above.

As the tectonic stress increases or as the available liquidity increases, the barrier to nucleation decreases. As long as the barrier is not infinite there is a non-zero probability that the first order nucleation event will occur. So if, for instance, the available liquidity increases for a time and then stays constant the system will have a barrier to nucleation that remains constant.

After some time, which depends on the size of the barrier, the nucleation event occurs. If we consider a system such as water, predicting this event would require knowing the position and velocities of all of the water molecules in the system. As with the case of earthquakes discussed above, this is simply not possible. This is also the case with financial markets. We would have to know the details of billions of transactions.

However, unlike systems in statistical mechanics and earthquakes where we know, through the laws of physics, exactly how the motion of one element of our system affects the others in financial markets there is considerable uncertainty even on the level of individual transaction. If, however, one is willing to live with forecasts (i.e. probabilities) rather than predictions then at least within the realm of statistical physics there is something that can be done. Probabilities can be calculated that will show how barriers to nucleation and lifetimes decrease with driving force. But to do this there must be an understanding of how the free energy depends on the driving force. (Gunton and Droz, 1983; Gunton et al, 1973) This has not been done for financial markets.

There is however one general trend in the understanding of meta-stability from statistical physics that might be applicable to financial markets. Since markets can be considered to be systems with long range interactions there will be a spinodal in such systems. The lifetimes of the metastable states in such systems are particularly long until one gets close to the spinodal where the decay of the metastable state becomes fairly probable.

But the spinodal is a critical point so that there are large fluctuations associated with being near the spinodal. For markets this translates into the idea that large fluctuations or high volatility are both associated with metastable states that are highly probable to undergo a first order transition in a relatively short time.(Klein et al., (2007).

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