Project 5

Physics 255 Econophysics

In this project, the goal is to write a computer code that will compute historical volatilities for your 10 time series, and then to compute the correlation coefficient between 1) the timeseries of 21 day trailing volatility, and 2) the time series of 12 month trailing returns, both computed from the original time series.

Note that the historical volatility for the SPX is *not* the VIX, although it is similar in some ways.

1. To compute the historical volatility we use the closing prices of the time series as for Project 3, $P(t_i)$ on trading day i. Then the historical volatility $V(t_i)$ on trading day i will be defined by:

$$V(t_i) = \sqrt{\frac{252}{21}\left(\sum_{j=1}^{21}\left(Log\left(\frac{P(t_{i-j})}{P(t_{i-j-1})}\right)\right)^2\right)}$$

Make sure you read this formula carefully:

a. You compute the ratio of the price on day *i* to the price on day *i*-1.
b. Then compute the natural Log of this ratio.
c. Then square it.
d. Then repeat a.-c. with the ratio of day *i*-1 to day *i*-2 and add to the term from a.-c.
e. Keep repeating this process until you have added up 21 terms over the last 21 trading days (counting today as the 1^{st} day – thus today is *i*=1).
f. Now divide by 21 to compute a ratio.
g. "Annualize" the result by multiplying by 252 trading days = 1 year.
h. Take the square root.

Compute and plot the volatilities for each of your 10 stocks/ETFs/securities.

2. The correlation coefficient ρ_{xy} of 2 time series, $x(t), y(t)$, is defined as follows:

$$\sigma_x^2 = \frac{1}{(t_2 - t_1)}\int_{t_1}^{t_2}(x(t) - \langle x \rangle)^2 dt$$

$$\sigma_y^2 = \frac{1}{(t_2 - t_1)}\int_{t_1}^{t_2}(y(t) - \langle y \rangle)^2 dt$$

$$\rho_{xy} = \left(\frac{1}{\sigma_x \sigma_y}\right)\left[\frac{1}{(t_2 - t_1)}\int_{t_1}^{t_2}(x(t) - \langle x \rangle)(y(t) - \langle y \rangle) dt\right]$$

$$\langle x \rangle = \frac{1}{(t_2 - t_1)} \int_{t_1}^{t_2} x(t)\ dt, \quad \langle y \rangle = \frac{1}{(t_2 - t_1)} \int_{t_1}^{t_2} y(t)\ dt$$

Note that ρ_{xy} is just a single number.

Perhaps to put it more simply:

Every day there is a $V(t)$ value calculated from the trailing 21 days of data in the timeseries $T(t)$.

Also, every day there is a $R(t)$ value calculated from the trailing 252 trading days (1 year).

So to compute the correlation, you start at day t=253, so there is both a value of $R(t)$ and a value of $V(t)$.

$R(t)$ is computed as $R(t) = (T(t) - T(t\text{-}252)) / T(t\text{-}252)$.

You then carry out the integral that defines ρ_{xy}, where $x(t)$ is $V(t)$, and $y(t)$ is $R(t)$.

Assignment:

a) Compute the 12-month trailing returns for each of your 10 time stocks/ETFs/securities. Also, compute the 21-day correlation coefficient for each of the pairs: (volatility, returns) for the 10 time stocks/ETFs/securities. Since the time series have different lengths, you will have to truncate the beginning of the volatility time series to have the same length as the returns time series.

b) What do you notice about the computed correlation coefficients and their relationship to the returns data?